Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced

Friday 28 June 2010 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 6 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. At time $t = 0$, the position vector of a particle P is $-3\mathbf{j}$ m. At time t seconds, the position vector of P is $\mathbf{r}$ metres and the velocity of P is $\mathbf{v}$ m s^{-1}. Given that

$$\mathbf{v} - 2\mathbf{r} = 4e^{t}\mathbf{j} \text{ ,}$$

find the time when P passes through the origin.

(7)

2.

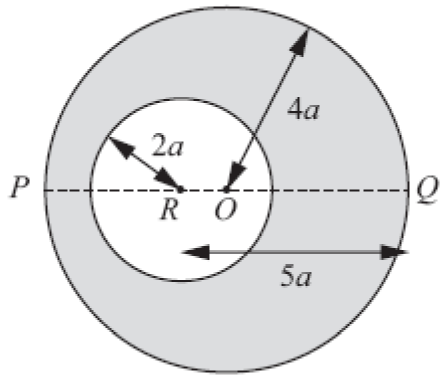

Figure 1

A uniform circular disc has mass $4m$, centre O and radius $4a$. The line POQ is a diameter of the disc. A circular hole of radius $2a$ is made in the disc with the centre of the hole at the point R on PQ where $QR = 5a$, as shown in Figure 1.

The resulting lamina is free to rotate about a fixed smooth horizontal axis L which passes through Q and is perpendicular to the plane of the lamina.

(a) Show that the moment of inertia of the lamina about L is $69ma^2$.

(7)

The lamina is hanging at rest with P vertically below Q when it is given an angular velocity Ω. Given that the lamina turns through an angle $\dfrac{2\pi}{3}$ before it first comes to instantaneous rest,

(b) find Ω in terms of g and a.

(6)

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3. A uniform lamina ABC of mass m is in the shape of an isosceles triangle with $AB = AC = 5a$ and $BC = 8a$.

(*a*) Show, using integration, that the moment of inertia of the lamina about an axis through A, parallel to BC, is $\frac{9}{2}ma^2$.

(6)

The foot of the perpendicular from A to BC is D. The lamina is free to rotate in a vertical plane about a fixed smooth horizontal axis which passes through D and is perpendicular to the plane of the lamina. The lamina is released from rest when DA makes an angle α with the downward vertical. It is given that the moment of inertia of the lamina about an axis through A, perpendicular to BC and in the plane of the lamina, is $\frac{8}{3}ma^2$.

(*b*) Find the angular acceleration of the lamina when DA makes an angle θ with the downward vertical.

(8)

Given that α is small,

(*c*) find an approximate value for the period of oscillation of the lamina about the vertical.

(2)

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4. Two forces $\mathbf{F}_1 = (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ N and $\mathbf{F}_2 = (3\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ N act on a rigid body.

The force $\mathbf{F}_1$ acts through the point with position vector $(2\mathbf{i} + \mathbf{k})$ m and the force $\mathbf{F}_2$ acts through the point with position vector $(\mathbf{j} + 2\mathbf{k})$ m.

(*a*) If the two forces are equivalent to a single force $\mathbf{R}$, find

(i) $\mathbf{R}$,

(2)

(ii) a vector equation of the line of action of $\mathbf{R}$, in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$.

(6)

(*b*) If the two forces are equivalent to a single force acting through the point with position vector $(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ m together with a couple of moment $\mathbf{G}$, find the magnitude of $\mathbf{G}$.

(5)

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5. A raindrop falls vertically under gravity through a cloud. In a model of the motion the raindrop is assumed to be spherical at all times and the cloud is assumed to consist of stationary water particles. At time $t = 0$, the raindrop is at rest and has radius a. As the raindrop falls, water particles from the cloud condense onto it and the radius of the raindrop is assumed to increase at a constant rate λ. A time t the speed of the raindrop is v.

(*a*) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{3\lambda v}{(\lambda t + a)} = g.$$

(8)

(*b*) Find the speed of the raindrop when its radius is $3a$.

(7)

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6. A uniform circular disc has mass m, centre O and radius $2a$. It is free to rotate about a fixed smooth horizontal axis L which lies in the same plane as the disc and which is tangential to the disc at the point A. The disc is hanging at rest in equilibrium with O vertically below A when it is struck at O by a particle of mass m. Immediately before the impact the particle is moving perpendicular to the plane of the disc with speed $3\sqrt{(ag)}$. The particle adheres to the disc at O.

(*a*) Find the angular speed of the disc immediately after the impact.

(5)

(*b*) Find the magnitude of the force exerted on the disc by the axis immediately after the impact.

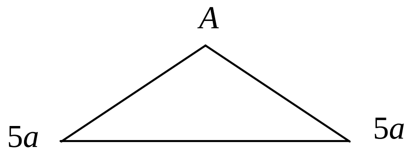
(6)

TOTAL FOR PAPER: 75 MARKS

END

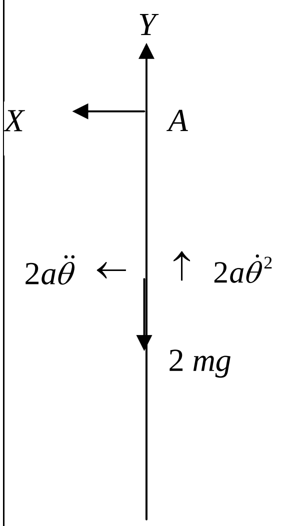
June 2010
6681 Mechanics M5
Mark Scheme

Question Number	Scheme	Marks
1	$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} - 2\mathbf{r} = 4\mathrm{e}^{t}\,\mathbf{j}$	
	$\text{IF} = \mathrm{e}^{-2t}$	
	$\mathrm{e}^{-2t}(\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} - 2\mathbf{r}) = \mathrm{e}^{-2t}.4\mathrm{e}^{t}\,\mathbf{j}$	M1
	$\frac{\mathrm{d}(\mathbf{r}\mathrm{e}^{-2t})}{\mathrm{d}t} = 4\mathrm{e}^{-t}\,\mathbf{j}$	
	$\mathbf{r}\mathrm{e}^{-2t} = \int 4\mathrm{e}^{-t}\,\mathbf{j}\,\mathrm{d}t$	DM1
	$= -4\mathrm{e}^{-t}\,\mathbf{j}\ (+\,\mathbf{C})$	A1
	$t = 0, \mathbf{r} = -3\mathbf{j} \Rightarrow \mathbf{C} = \mathbf{j}$	DM1
	$\mathrm{e}^{-2t}\mathbf{r} = (1 - 4\mathrm{e}^{-t})\mathbf{j}$ or $\mathbf{r} = (\mathrm{e}^{2t} - 4\mathrm{e}^{t})\mathbf{j}$	A1
	$(1 - 4\mathrm{e}^{-t}) = 0$ or $(\mathrm{e}^{2t} - 4\mathrm{e}^{t}) = 0$	DM1
	$t = \ln 4$, 1.4 or better	A1
		7
2 (a)	Mass of disc removed = m	B1
	$\frac{1}{2}4m(4a)^2 + 4m(4a)^2$	M1 A1
	$\frac{1}{2}m(2a)^2 + m(5a)^2$	M1 A1
	$I = \frac{1}{2}4m(4a)^2 + 4m(4a)^2 - (\frac{1}{2}m(2a)^2 + m(5a)^2)$	DM1
	$= 69ma^2$ *	A1
		(7)
(b)	$4m.0 = 3m\bar{x} - ma$	M1
	$\bar{x} = \frac{1}{3}a$ (from O)	A1
	$\frac{1}{2}69ma^2\Omega^2 = 3mg(4a - \frac{1}{3}a)(1 - \cos\frac{2\pi}{3})$	M1 A2
	$\Omega = \sqrt{\frac{11g}{23a}}$	A1
		(6) **13**

Question Number	Scheme	Marks
3		
(a)	$\delta A = \frac{8x}{3}\delta x$	M1 A1
	$\delta m = \frac{8x}{3}\delta x.\frac{m}{12a^2}$ or $\delta m = \frac{8x}{3}\delta x.\rho$	DM1
	$\delta I = \frac{8x}{3}\delta x.\frac{m}{12a^2}x^2 \quad (= \frac{2m}{9a^2}x^3\delta x)$	A1
	$I = \int_0^{3a}\frac{2m}{9a^2}x^3 \, \mathrm{d}x$	M1
	$= \frac{2m}{9a^2}\left[\frac{x^4}{4}\right]_0^{3a}$	
	$= \frac{9ma^2}{2}$*	A1
		(6)
(b)	$I_A = \frac{9ma^2}{2} + \frac{8ma^2}{3} = \frac{43ma^2}{6}$ (perp axes rule)	M1 A1
	$I_A = I_G + m(2a)^2$ (parallel axes rule)	DM1 A1
	$I_D = I_G + ma^2$ (parallel axes rule)	A1
	$I_D = \frac{43ma^2}{6} - 3ma^2 = \frac{25ma^2}{6}$	A1
	$mga\sin\theta = -\frac{25ma^2}{6}\ddot{\theta}$	M1
	$\ddot{\theta} = -\frac{6g}{25a}\sin\theta$	A1
		(8)
(c)	For small θ, $\ddot{\theta} = -\frac{6g}{25a}\theta$ SHM	M1
	$T = 2\pi\sqrt{\frac{25a}{6g}} = 5\pi\sqrt{\frac{2a}{3g}}$	A1
		(2) **16**

Question Number	Scheme	Marks
4 **(a) (i)**	$\mathbf{R} = (\mathbf{i}+2\mathbf{j}+3\mathbf{k})+(3\mathbf{i}+\mathbf{j}+2\mathbf{k})$	M1
	$=(4\mathbf{i}+3\mathbf{j}+5\mathbf{k})$	A1
		(2)
(a) (ii)	$(x\mathbf{i}+y\mathbf{j}+z\mathbf{k})\text{x}(4\mathbf{i}+3\mathbf{j}+5\mathbf{k}) = (2\mathbf{i}+\mathbf{k})\text{x}(\mathbf{i}+2\mathbf{j}+3\mathbf{k})+(\mathbf{j}+2\mathbf{k})\text{x}(3\mathbf{i}+\mathbf{j}+2\mathbf{k})$	M1
	$(5y-3z)\mathbf{i}+(4z-5x)\mathbf{j}+(3x-4y)\mathbf{k} = (-2\mathbf{i}-5\mathbf{j}+4\mathbf{k})+(6\mathbf{j}-3\mathbf{k})$	A2
	$(5y-3z)\mathbf{i}+(4z-5x)\mathbf{j}+(3x-4y)\mathbf{k} = (-2\mathbf{i}+\mathbf{j}+\mathbf{k})$	
	a solution is $x=0, y=-\frac{1}{4}, z=\frac{1}{4}; x=\frac{1}{3}, y=0, z=\frac{2}{3}; x=-\frac{1}{5}, y=-\frac{2}{5}, z=0$	B1
	$\mathbf{r} = -\frac{1}{4}\mathbf{j}+\frac{1}{4}\mathbf{k}+\lambda(4\mathbf{i}+3\mathbf{j}+5\mathbf{k})$	M1 A1 ft
		(6)
(b)	$(\mathbf{i}+2\mathbf{j}+\mathbf{k})\text{x}(4\mathbf{i}+3\mathbf{j}+5\mathbf{k})+\mathbf{G} = (-2\mathbf{i}+\mathbf{j}+\mathbf{k})$	M1 A1
	$(7\mathbf{i}-\mathbf{j}-5\mathbf{k})+\mathbf{G} = (-2\mathbf{i}+\mathbf{j}+\mathbf{k})$	
	$\mathbf{G} = (-9\mathbf{i}+2\mathbf{j}+6\mathbf{k})$	A1
	$\lvert\mathbf{G}\rvert = \sqrt{(-9)^2+2^2+6^2}$	M1
	$=11$ (Nm)	A1 ft
		(5)
		13

Question Number	Scheme	Marks
5		
(a)	$\frac{\mathrm{d}r}{\mathrm{d}t} = \lambda \Rightarrow r = \lambda t + a$	B1
	$(m+\delta m)(v+\delta v) - mv = mg\delta t$	M1 A1
	$\frac{\mathrm{d}v}{\mathrm{d}t} + \frac{v}{m}\frac{\mathrm{d}m}{\mathrm{d}t} = g$	DM1 A1
	$\frac{\mathrm{d}m}{\mathrm{d}t} = 4\pi r^2(\rho)\lambda$	A1 (B1)
	$\frac{\mathrm{d}v}{\mathrm{d}t} + \frac{3v}{4\pi r^3\rho}4\pi r^2\rho\lambda = g \Rightarrow \frac{\mathrm{d}v}{\mathrm{d}t} + \frac{3v\lambda}{r} = g$	DM1
	$\frac{\mathrm{d}v}{\mathrm{d}t} + \frac{3v\lambda}{\lambda t + a} = g$ *	A1
		(8)
(b)	$R = \mathrm{e}^{\int \frac{3\lambda}{\lambda t + a}\mathrm{d}t} = \mathrm{e}^{3\ln(\lambda t + a)} = \mathrm{e}^{\ln(\lambda t + a)^3} = (\lambda t + a)^3$	M1 A1
	$v(\lambda t + a)^3 = g\int(\lambda t + a)^3\mathrm{d}t$	DM1
	$v(\lambda t + a)^3 = \frac{1}{4\lambda}g(\lambda t + a)^4$	A1
	$t = 0, v = 0 \Rightarrow C = -\frac{1}{4\lambda}ga^4$	DM1
	$\lambda t + a = 3a$	DM1
	$v = \frac{1}{4\lambda}g(3a) - \frac{1}{4\lambda}\frac{ga^4}{27a^3} = \frac{20ag}{27\lambda}$	A1
		(7) **15**

Question Number	Scheme	Marks
6		
(a)	MI of disc about $L = \frac{1}{4}m(2a)^2 + m(2a)^2 = 5ma^2$	M1 A1
	CAM: $m3\sqrt{ag}.2a = (5ma^2 + m(2a)^2)\omega$	M1 A1 ft
	$\omega = \frac{2}{3}\sqrt{\frac{g}{a}}$	A1
		(5)
(b)		
	$M(A),\ 0 = I\ddot{\theta}$	B1
	$\ddot{\theta} = 0$	
	$R(\leftarrow), X = 2m2a\ddot{\theta} = 0$	B1
	$R(\uparrow), Y - 2mg = 2m2a\dot{\theta}^2$	M1 A1
	$Y = 2mg + 4ma\frac{4g}{9a}$	DM1
	$= \frac{34mg}{9}$	A1
		(6) **11**